I. INTRODUCTION

Did I see it go by, 
That Millikan mote? 
Well, I said that I did. 
I made a good try. 
But I’m no one to quote. 
If I have a defect 
It’s a wish to comply 
And see as I’m bid. 
I rather suspect 
All I saw was the lid 
Going over my eye. 
I honestly think 
All I saw was a wink. 

—Robert Frost, *A Wish to Comply*

Michael Faraday was the first investigator to attempt to discover the magnitude of the fundamental electric charge. His experiments on electrolysis showed that a given quantity of electricity, the faraday (about 96,500 C), was required to decompose one mole of monovalent ions. Faraday understood that a mole was a fixed number of particles, and that if one knew Avogadro’s number, one could calculate the value of the fundamental charge from the value of the faraday and this number. (Equivalently, if one knew the value of the fundamental charge, one could determine the value of Avogadro’s number.) Unfortunately, Faraday did not know either number. In 1874, G. J. Stoney, using an estimate of $N_A$ from kinetic theory, estimated the value of $e$ to be $\approx 10^{-20}$ C. The next attempt to measure the value of $e$ was undertaken by J. S. Townsend, a student of J. J. Thompson. Thompson had just finished his work to measure the value of $e/m$, and this was a natural next step. The method that he employed, which involved measuring the total mass and charge of a cloud made from gaseous ions and water, had many uncertainties that were difficult to solve, and never produced a satisfactory result. R. M. Millikan modified this method; he attempted to use an electric field to hold the cloud in place, thus allowing him to continuously monitor it. He found that this was not successful, but he did discover that he could hold individual drops stationary between two metal plates by adjusting the field appropriately. This discovery ultimately led to the famous Millikan oil-drop experiment, in which Millikan observed oil drops between two metal plates as they rose and fell under the action of electric fields and gravity. Millikan would measure the motion of one drop for several hours at a time, and from his measurements could deduce the charge on the individual drops. He found that the charges always occurred in integral multiples of a fundamental charge, as expected. Most importantly, he was able to determine the value of this fundamental charge to an accuracy of 1 part in 1000. (It was subsequently found that this number was in error by four times this amount due to an inaccurate value of the viscosity of air that had been used.)

Millikan performed his experiments in the years 1909–1913, and received the Nobel Prize for his efforts.

You will perform your experiments in two four-hour lab sessions; you will most likely not receive any prize for your efforts, but you will have the satisfaction of replicating a classic experiment and making a reasonably accurate determination of one of the basic fundamental constants.

II. THEORY

The basic experimental setup is very simple. Two metal plates of a known separation $D$ are surrounded by a collar to form a closed chamber. Using an atomizer, oil droplets are injected into the space between the two plates; the droplets are charged by this action. Typical droplet diameters are on order of about 1 $\mu$m (10$^{-6}$ m). For such small droplets, surface tension assures that the droplets are essentially perfect spheres.

The droplets will fall slowly under the influence of gravity unless a suitable voltage applied between the plates, leading to an electric field between them. If a droplet is charged, this field results in an electric force on the drop which points up or down, depending on the direction of the field. Thus the voltage (and hence the field) can be adjusted so that the droplets rise and fall with any desired speed. A careful adjustment of the field will allow you to keep a single droplet in view for quite a long time. The drops are backlight by an intense light source; the scattered light from the drops renders them visible as small points of light, which used to be known poetically as “Millikan’s Shining Stars”.

When the droplet is in motion, there is a third force present in addition to those of gravity and the electric field. A moving drop will feel a drag force due to air resistance; this force is proportional to the speed of the drop. In the regime considered here, with very small droplets moving in air, the magnitude of the drag force can be accurately described by Stokes’s Law,

$$F_d = 6\pi \eta rv,$$

where $\eta$ is the viscosity of the air, $r$ the radius of the drop, and $v$ its speed. The dependence of this force on velocity and the small mass of the sphere mean that the spheres will reach terminal velocities nearly instantaneously. You should verify that the time constant is so small that for the measurement times you will be using, the sphere may be considered to reach terminal velocity immediately.

With the three forces above—gravity, the electric force, and the drag force—a free-body diagram for an oil drop being...
pulled upward by an electric field can be constructed as in Fig. 2(a). Again, the drag force quickly rises until the drop moves at a constant upward speed \( v_u \). With the speed constant, the acceleration is zero and Newton’s second law yields

\[
F = qE - mg - F_d = ma = 0,
\]

or

\[
qE = mg + F_d = mg + 6\pi \eta rv_u. \tag{2}
\]

Now we can look up values of the acceleration of gravity \( g \) and the viscosity of air \( \eta \), and we measure experimentally values for the electric field \( E \) and the (upward) velocity of the drop \( v_u \). To find the charge \( q \), however, we still need to find the drop mass \( m \) and its radius \( r \). In fact, because these two are related by \( m = \rho 4\pi r^3/3 \), where \( \rho \) is the density of the oil, what we really need is a way to measure \( r \).

Since the droplets are so small, it is impossible to make a direct measurement of their radii. Instead, we can turn Stokes’s law to our advantage by using it to measure \( r \) by considering the motion of the drop as it falls without an applied electric field. Then the free-body diagram [Fig. 2(b)] yields from Newton’s Law

\[
mg = 6\pi \eta rv_d. \tag{3}
\]

Using the above relation between \( m \) and \( r \), we can solve this for the radius, yielding

\[
r = \sqrt{\frac{9\eta v_d}{2\rho g}}. \tag{4}
\]

Inserting Eqs. 3 and 4 into Eq. 2 allows us to solve for the charge \( q \) on the drop.

### III. PROCEDURE

#### A. Setup

The apparatus that you will use to perform the experiment is a one-piece unit; the power supply, polarity-reversing switch, light source, microscope, chamber, atomizer and oil reservoir are all contained in one unit. A small video camera has been attached to the microscope to allow recording of videos of the drops’ motion to the computer. The “Grabbee” software or Windows Video Maker can be used to make such recordings. The videos can then be analyzed using Movie Maker or ImageJ.

Note that the light source is focused to make a very bright area in the center of the chamber. This is also at the approximate center of travel of the telescope focuser. Do not make any adjustments to the light source unless you are certain that there is a problem and you have checked with the instructor.

Place the end of the atomizer tube in the reservoir. Close the chamber and turn on the unit. Set the voltage to zero. A gentle squeeze of the bulb should inject some droplets into the region between the plates. You will see a bright flash as the drops enter the chamber. You’ll need to replace the plug into the inlet hole to keep air currents from interfering with the drops’ motion. Note that the field of view of the telescope is somewhat limited; while you are viewing a single drop you will find it necessary to move the focus forward or backward in order to follow its motion. In order to find a drop to follow you also have to move the focus back and forth; it is rare that a squeeze of the atomizer will inject a drop right in your field of view, but by looking at different depths you should be able to find one.

Once you have found a drop, explore your ability to move it up and down using the electric field. Vary the voltage and polarity; you will see drops of opposite charges move in opposite directions. Drops with larger charges will move more rapidly. The basic goal of this experiment is to determine the size of the fundamental charge, so it will be necessary and desirable for you to find spheres with small charges—somewhere in the range from one to about eight electron charges. As noted above, the basic measurement that you will have to make on an individual drop is a measurement of the speed when the drop is driven upwards by an electric field, and then allowed to fall without a field. To find this speed, you need to calibrate how many video pixels correspond to 1 mm. For this purpose, there is a calibration block with two lines scribed accurately 1.000 ± 0.003 mm apart.

As you are determining a strategy for taking your data, note the following:

i) You need to concentrate on small drops which have only a few charges on them. You need drops with only a few charges on them so that you can distinguish between charge states. For instance, suppose your measurement error is 10%. Then it is easy to distinguish between \( q = 2e \) and \( q = 3e \), but \( q = 10e \) and \( q = 11e \) are much harder to tell apart. Small numbers of charges mean that the upward force \( qE \) will be small, so we also need to have small, light drops so that \( qE \) can be made larger than \( mg \) and the drops can be made to rise. To find small, lightly-charged drops to work with it can be helpful to first select droplets which fall rather slowly in zero field; typical fall times for good drops are in the range of 10 to 30 s to fall about 2 mm. After you get a handle on how a small drop falls, you can check for ones which also have only a few charges by applying a fairly large voltage (≈ 200–500 V) and seeing which droplets rise at speeds similar to those of
the smaller falling droplets. Droplets which rise very rapidly probably have too much charge to be useful.

ii) Repeated measurements on a single drop are very useful. There are two benefits: first, you will be able to estimate the error in your determinations of speed. Second, you will be able to get a better value for the charge on the drop by averaging the measurements that you obtain. Therefore, the best strategy is to work with a single drop for at least several (~3–4) measurements. It is not difficult to make measurements on a single drop for a few minutes at a time; this will belong enough for several measurements of speed up and speed down.

B. Analysis and Things to Consider

The basic analysis that you should make on your data is to determine the charge that was present on each of the drops that you measured. You should make this calculation as you go; this will tell you how accurately you are working, and give you ideas as to how to improve your data collection procedure. Using a spreadsheet such as Excel is a very useful way to collect your data, as it can be programmed to immediately give the charge on each droplet, without the tedious calculations which are necessary if doing it by hand.

When presenting your data, you should do so in a graphical form that shows that the charges that you have measured appear in integral multiples of a fundamental charge. To do this convincingly, you will have to take many data points. Balance this demand against the need to collect lengthy data on single drops; you should establish a criterion for determining when you have taken enough data on one drop and should move on. Time will be a real limitation in this experiment, and you should budget yours wisely.

Finally, you should combine your data from all of your results into one measurement of, the fundamental charge, with an error estimate. This is the most crucial outcome of the experiment, even historically. The quantization of charge had been expected, though its verification was nice. But it was the actual measurement of the value of \( e \) that led to the Nobel Prize.

C. Advanced Analysis

To get the best possible data, one needs to make a correction to Stokes’s law. Stokes’s law holds for spheres moving in fluids with very low Reynolds’ Numbers. For a micron-sized sphere in air, the Reynolds’ number is very small, and Stokes’s law should work very well. However, in this experiment the drops are so small that they are beginning to approach the mean free path of the air molecules. Thus the physics is not precisely that of a sphere moving in a continuous fluid; instead, one must begin to think in terms of a molecular model wherein the sphere is being constantly struck by air molecules. It is possible to still use Stokes’s law if we use a modified viscosity for air. This modified viscosity should depend on the air pressure (the ordinary viscosity does not), because when the pressure is low the mean free path, and thus the correction to Stokes’s law, becomes large. From the above discussion, we see that it should also depend on the radius of the drop. It turns out that the viscosity to use in Stokes’s law is

\[
\eta = \eta_0 \left[1 + \frac{5.908 \times 10^{-5}}{rP}\right]^{-1}.
\]

Here, \( \eta_0 \) is the “ordinary” viscosity of air, \( P \) is the air pressure in torr (mm of mercury), and \( r \) is the drop radius in meters. Notice that if either \( r \) or \( P \) are large, no correction to Stokes’s law is necessary, as expected. The viscosity of air depends on temperature (but not on pressure). Near room temperature, this viscosity can be accurately written as

\[
\eta_0 = 1.8228 \times 10^{-5} + (4.790 \times 10^{-8})(T - 21^\circ C)
\]

where \( T \) is in celsius and the viscosity is in N·s/m².

The difficulty in applying this correction to \( \eta \) is that it contains \( r \). However, we found \( r \) by using Stokes’s law, which involves \( \eta \)! The easiest way around this is to first calculate \( r \) without using the correction. Then use this uncorrected value of \( r \) to find the corrected value of \( \eta \) from Eqn. 5 above. Then use this new value of \( \eta \) in Stokes’s law to find a better value for \( r \). You can repeat this cycle as many times as necessary, but typically after one iteration your values of \( \eta \) and \( r \) will converge nicely.

D. Even More Advanced Analysis

Equation 5 contains the factor \( 5.908 \times 10^{-5} \). Where did this come from? It was Millikan who, in the course of his
oil drop experiment, first realized that Stokes’s law needed modification when the drops were very small. He noticed that the charge deduced from an application of the uncorrected Stokes’s was not perfectly quantized—the values of the charge varied from drop to drop. He soon realized that the values of the charges were correlated with the radii of the drops, and this led him to consider the idea that Stokes’s law needed modification for small radius drops. Stokes’s law assumes a spherical object moving through a continuous fluid. But for the smallest drops we’ll study, the assumption of a continuous fluid is not quite correct because the mean free path \( \lambda \) of the air molecules is starting to approach the size of the drop (at 23°C and 760 torr, \( \lambda \) is about 0.0673 \( \mu \)m).

These observations suggest that we write a corrected Stokes’s law in the form

\[
F_d = 6\pi \eta_0 rv + f\left(\frac{\lambda}{r}\right)
\]

where \( \eta_0 \) is the viscosity of air in the limit where \( \lambda \ll r \), that is, the value given by Eq. 6.

The function \( f \) contains the correction to Stokes’s law. Note that the argument of \( f \) is \( \lambda/r \), which goes to zero in the limit that \( \lambda \ll r \). This means that \( f(\lambda/r) \) must go to 0 in this limit so that the original form of Stokes’s law is recovered. For the drops we’ll study, the ratio \( \lambda/r \) is small, so it makes sense to expand \( f \) in a Taylor’s series as

\[
f(\lambda/r) = A\lambda/r + B(\lambda/r)^2 + \cdots
\]

Keeping only the lowest order term, we can write Stokes’s law as

\[
F_d = \frac{6\pi \eta_0 rv}{1 + A\lambda/r} = 6\pi \eta rv
\]

where \( \eta = \eta_0/(1 + A\lambda/r) \) can be thought of as the viscosity corrected for the case where the mean free path is not negligible compared to the drop radius.

Millikan knew that when the drop radius \( r \) was very large compared to the mean free path \( l \) of the air molecules, there would be no correction needed, and Eqn. 1 would work perfectly. Only when \( r \) became small, and/or \( l \) large, would a correction be required. Thus a corrected form of Stokes’s law can be written

\[
F_d = 6\pi \eta(r,l)rv,
\]

where the effective viscosity \( \eta(r,l) \) must limit to the ordinary viscosity \( \eta_0 \) when \( r \) became small, or \( l \) large. Thus we can expand \( \eta \) in a power series in the small parameters \( 1/r \) and \( l \), or equivalently, \( 1/r \) and \( 1/P \), where \( P \) is the pressure. When \( P \) is large, \( l \) is small, and so \( 1/P \) is an appropriate variable for expansion. Thus \( \eta(r,P) \) can be expanded as

\[
\eta(r,P) = \eta_0[1 + b/rP + c/(rP)^2 + \ldots]
\]

where \( b, c, \ldots \) are constants. Notice how this expression limits to \( \eta_0 \) when \( r \) and/or \( P \) are large, as it must. Because the correction was known to be small, Millikan retained only the first two terms in the expansion (so \( c = 0 \)).

However, there is still the unknown constant \( b \); this is the value \( 5.908 \times 10^{-5} \) used above. How can this be determined? To see how he did it, read his original papers; they are masterpieces of scientific discovery!

### E. Data

The following data about the equipment will be needed:

- Distance between metal plates in chamber: 4.902±0.007 mm.
- Viscosity of air: See Fig. 2 and Eq. 6.
- Density of oil (Nye’s watch oil): 0.861±0.001 g/cm³.