The Hall Effect

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Abstract

The Hall effect in a thin bismuth strip was investigated. The Hall voltage was found to be a linear function of the applied current, in agreement with theoretical predictions. However, the Hall voltage was a highly nonlinear function of the magnetic field, in contrast to the linear form predicted by the semiclassical theory. At high magnetic fields, where electron scattering is less important and the simple theory is expected to hold, we find an electron density of around $1–2 \times 10^{19}$ cm$^{-3}$.

1 Introduction

When a current-carrying conductor is placed in a magnetic field perpendicular to the current direction, a voltage develops transverse to the current. This voltage was first observed in 1879 by Edwin Hall during the course of his studies on the effect of a magnetic field on the current path through a conductor. Hall believed that the field would push on the current in such a way as to “squeeze” it to one side, effectively reducing the width of the conductor and increasing its resistance. Maxwell, however, understood that there would be no change in the current path, essentially because compression of the electron “fluid” would lead to a huge charge imbalance, with resulting electric fields which exactly countered the force of the magnetic field. Experimentally, Hall found that the ordinary resistance was in fact independent of field, but he discovered that a voltage—today called the Hall voltage—developed across the width of the sample. We now understand this transverse voltage as a consequence of Maxwell’s idea: it is the electric fields that balance the magnetic force that lead to the Hall voltage.

The Hall effect has since lead to a deeper understanding of the details of the conduction process. In particular, as we will see here, a quantitative application of the Hall effect in a conducting medium can yield the density of the charge carriers as well as their sign. In simple metals, the density of carriers is an integer multiple of the density of atoms, and their sign is found to be negative; this indicates that each atom donates a fixed number of electrons to the conduction process. However, in more complex metals (such as the bismuth studied here) or particularly in semiconductors, the carrier density is found to be much lower than one per atom. And, remarkably, the sign of the carriers can even be positive! These anomalous Hall effects are explained by the band theory of solids, which predicts the possibility of only a small number of thermally-excited carriers, which may be holes with an effective positive charge.

2 Theory

Here we outline a simple semiclassical derivation of the Hall effect. Consider a current-carrying strip of width $w$ and thickness $t$, with the $x$-axis oriented in the direction of the current, the $y$-axis in the plane of the strip, and the $z$-axis perpendicular to the plane of the strip (Fig. 1). We now apply a uniform magnetic field
\( \mathbf{B} = B\mathbf{\hat{z}} \). In equilibrium, the \( y \)-component of the force on a mobile charge carrier (an electron or hole) in the conductor must be zero, since there can be no net current in the \( y \) direction in equilibrium. The force in this direction is due to the Lorentz force from the magnetic field, and to the \( y \)-component of the electric field. Thus we have
\[
F_y = qvB + qE_y = 0,
\]
where \( v \) is the drift velocity of a charge carrier, \( q \) is its charge (\( -e \) for electrons, \( +e \) for holes), and \( E_y \) the electric field component. Solving for the electric field yields
\[
E_y = -vB,
\]
giving a Hall voltage across the width of the strip
\[
V_H = E_y w = -vBw. \tag{1}
\]

The electron velocity \( v \) is not an experimentally accessible quantity. However, we note that if the electron density in the metal is \( n \) (electrons/cm\(^3\)), then the current density is \( J = nqv \), and the current is \( I = Jw t = nqvw t \) so that \( v = I/nqw t \). Inserting this into Eq. 1, we have
\[
V_H = -vBw = -BI/nqvt \equiv -R_H BI/t, \tag{2}
\]
where \( R_H = 1/nq \) is the Hall coefficient. This illustrates the important fact that, within the simple model given, the Hall voltage \( V_H \) is directly proportional to both the magnetic field \( B \) and the applied current \( I \). We also note that from a measurement of the Hall voltage at various values of \( B \) and \( I \), and knowing the sample thickness and the value of the electronic charge \( e \), we may deduce the carrier density \( n \). This simple experiment, then, probes the details of the electronic conduction process.

## 3 Experimental Details

The sample used was a thin strip of bismuth, with dimensions \( \ell = 1.55 \) cm, \( w = 0.83 \) cm, and \( t = 0.064 \) cm. Bismuth was chosen because it is a semimetal: it is essentially a semiconductor whose conduction and valence bands slightly overlap. This gives a very low density of conduction electrons. As we see from Eq. 2, a small value of \( n \) will yield a large value of \( V_H \) for a given field and current, which makes the measurement of \( V_H \) much simpler. Using an ordinary metal like silver, with its very large value of \( n \), would lead to very difficult-to-measure Hall voltages.
The bismuth strip was held between the poles of an electromagnet capable of providing fields between ±8 kG. Because of hysteresis in the magnetic field, the field was directly measured using a gaussmeter inserted along with the sample. Current through the strip was limited to ±1 A to avoid problems with heating. The Hall voltage was measured using a DVM with a resolution of 1 µV.

4 Results and Discussion

4.1 Determination of offset voltages

Two effects can add spurious offsets to the true Hall voltage. First, there is a possible small longitudinal (i.e., in the x-direction) misalignment of the two Hall contacts on the sample (Fig. 1). This leads to a spurious contribution to \( V_H \) due to the ordinary resistive voltage drop along the sample. Because this is an ordinary resistive voltage, we expect that it will be proportional to the current, but be independent of the magnetic field. Its value can thus most easily be measured at zero applied field, where the Hall contribution is zero. The second spurious contribution to the measured voltage arises from possible thermoelectric voltages which may arise when the two sample contacts are at different temperatures. Again, we expect this contribution to be field-independent, and again a zero-field measurement will allow its magnitude to be determined.

We show the results of such measurements in Fig. 2, where we plot the sample voltage as a function of applied current at a essentially zero applied field \( (B = 0.3 \pm 0.2 \text{ G}) \). The voltage is a nearly linear function of the current, as would be expected if the main contribution to the spurious voltage were due mainly to the contact misalignment. A polynomial fit to the data yields \( V_{\text{offset}}(I) = 0.59 - 196.51I - 0.168I^2 \) for \( I \) in amps and \( V \) in µV. The first and last terms represent the effects of the thermal voltage, which can generate a voltage even with no current (first term), and which might be expected to rise as \( I^2 \) (last term) due to \( I^2R \) Joule heating. These two terms are very small, always less than 1 µV for the range of currents studied. The main contribution is then the linear term, which corresponds to a misalignment resistance of \( R = V_H/I = 196.5 \mu\Omega \). This data can be used to correct our raw Hall data, by subtracting \( V_{\text{offset}}(I) \) from any datum taken at a current \( I \).

![Figure 2](image.png)

Figure 2: The measured voltage vs. current at zero applied field. With no applied field, there is no Hall effect, so this voltage is the spurious one due to misalignment and thermal offsets. This spurious voltage is subtracted from all subsequent measurements, yielding the true Hall voltage.

In Fig. 3 we show that this correction works quite well. The closed circles represent the uncorrected voltage obtained as a function of applied field \( B \) at a fixed current of 1.00 A. We note that there is a finite voltage measured at zero field, which is due to the offset voltage just discussed. By subtracting off \( V_{\text{offset}}(1.0 \text{ A}) \) from all the data, we obtain the curve given by the open circles. Since we have removed the spurious
offset voltage, we are left with the true Hall signal, which goes to zero when $B = 0$ as expected. In all data shown hereafter, this correction has been applied.

![Figure 3: Raw (uncorrected) data (black circles), and corrected (white circles) data after subtraction of the offset voltage from Fig. 2.](image3.png)

**4.2 Results and Analysis**

We examine first the behavior of the Hall voltage $V_H$ as the current $I$ is varied; we find that $V_H$ has the simple linear dependence on $I$ predicted by Eq. 2. In Fig. 4 the Hall voltage is plotted versus the current at three values of the magnetic field. At each field, the voltage rises quite linearly with current, and at higher fields the slope of the lines is greater. Equation 2 predicts this linear dependence of $V_H$ on $I$, and is basically a statement that the Lorentz force is proportional to the current.

![Figure 4: The Hall voltage vs. sample current at applied fields $B = 1.0$, $4.0$, and $8.0$ kG. The linear dependence is in accord with Eq. 2.](image4.png)

The dependence of $V_H$ on field is more complex. In Fig. 5 we plot $V_H$ versus applied field, at several values of the sample current. We notice that, unlike the $I_s$-dependence, the dependence of $V_H$ on $B$ is quite
nonlinear, with a steep initial slope which flattens off at higher fields. This nonlinearity is not in agreement with Eq. 2, which predicts a strictly linear dependence of $V_H$ on $B$.

![Graph showing Hall voltage $V_H$ as a function of applied field $B$ with sample currents of 0.25, 0.50, and 1.0 A.]

**Figure 5:** The Hall voltage $V_H$ as a function of applied field $B$, at sample currents of 0.25, 0.50, and 1.0 A. Note that the curves are quite nonlinear, with decreasing slopes at high fields.

We can further investigate this nonlinearity by plotting the Hall coefficient as defined in Eq. 2, from which we see

$$R_H = \frac{t}{I} \frac{\partial V_H}{\partial B}. \quad (3)$$

In Fig. 6 we plot the Hall coefficient $R_H$ calculated in this way. The derivative in Eq. 3 was calculated by fitting a spline to the $I_s = 1.0$ A data of Fig. 5, and taking its numerical derivative. We see that the Hall coefficient varies by more than a factor of two from $B = 0$ to the highest fields. Interestingly, the data is also asymmetrical in field as well. $R_H$ may be compared to values in the literature. Kittel [1] gives a value of $R_H = 5 \times 10^{-9} \Omega \text{cm}/\text{G}$, while Hoffman [2] gives the rather smaller value of $0.7 \times 10^{-9} \Omega \text{cm}/\text{G}$. From Fig. 6 we see that our value of $R_H$ near zero field is somewhat large compared to these values, about $15 \times 10^{-9} \Omega \text{cm}/\text{G}$. However, it does fall at higher fields to the $4-6 \times 10^{-9} \Omega \text{cm}/\text{G}$ range, similar to the value of Kittel.

![Graph showing Hall coefficient $R_H$ and carrier density as a function of magnetic field.]

**Figure 6:** The Hall coefficient (left axis) and the carrier density (right axis) as a function of magnetic field.
These discrepancies, and the nonlinearity of the Hall coefficient, may be explained by a more detailed theory of the Hall effect, which properly includes the shape of the Fermi surface of the metal under study. This more detailed theory shows that the value Hall coefficient at low fields depends in a complex way on the amount of scattering in the sample, that is, on its purity and overall microscopic crystallite structure. These details can vary significantly between samples, and might explain the differences between values found by different authors. The detailed theory also indicates that the Hall effect can be quite different at high fields than at low fields. At high magnetic fields the carriers move in tight circular orbits; when the radius of these orbits becomes less than the scattering length, the details of the scattering become unimportant and the Hall coefficient should saturate at a value given by the simple relation $R_H = 1/ne$. We do note that it is our high field values which agree more closely with the value of Kittel.

From $R_H$ it is possible to calculate the carrier density in the bismuth sample. From Eq. 2 we have $n = 1/eR_H$, which is also plotted in Fig. 6. At high fields, where this relation is more accurate, we find a carrier density of about $1–2 \times 10^{19}/\text{cm}^3$. It is interesting to compare this to the number density of atoms $n_A$ in bismuth. We have $n_A = N_A \rho/W_A$, where $W_A$ is the atomic weight, $N_A$ is Avogadro’s number, and $\rho$ is the density. Putting in the appropriate values for bismuth we find $n_A = 2.8 \times 10^{22}/\text{cm}^3$, a value some 2000 times higher than our calculated carrier density. Again, this can be explained from band theory. Unlike a normal metal, in which the conduction band is filled with an integral number of electrons, bismuth has a conduction band which normally would be empty, as in a semiconductor. However, unlike a true semiconductor there is a small overlap with the valence band, which then contributes a very small number of electrons to the conduction band, yielding the very low carrier density observed.

Finally, we can calculate the sign of the carriers. Care was taken to carefully record the directions of the magnetic field and the current, and the polarity of the resulting voltage. In terms of Fig. 1 we defined a positive field to be in the $z$-direction, and a positive current in the $x$-direction. With such a field and current applied, the resulting voltage implied an electric field in the $+y$ direction. The minus sign in Eq. 2 shows that when all three quantities $B$, $I$, and $V_H \propto E_y$ are positive, as is experimentally the case, the charge $q$ must be negative. The the carriers in bismuth are electrons, not holes.

5 Conclusions

We have explored the Hall effect in bismuth, and found experimentally that there are significant deviations from the simple classical theory. A correct quantum band theory must be invoked to at least qualitatively explain these discrepancies. The carrier density and sign can be obtained from these experiments, and we find a very low density of electrons, consistent with the semimetallic nature of bismuth.

References
